647510 **BSCgraCC6010** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination

March/April -2020

GRAPH THEORY AND COMPLEX ANALYSIS -2 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

1(A). Answer any two out of three. **[10]**

(1) Define Complete graph. Prove that in complete graph with n vertices there are $\frac{n-1}{2}$ edge disjoint Hamiltonian circuits.

(2) State and prove necessary and sufficient condition, for a graph being Euler graph.

(3) Prove that a simple graph with n vertices and k-components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

1(B). Answer any one out of two. **[4]**

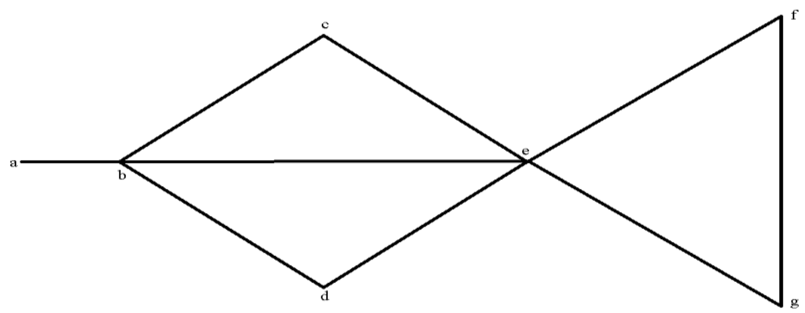
(1) Prove that number of vertices of odd degree in a graph is always even.

(2) Define: Hamiltonian circuit. Prove that number of pendant vertex in binary tree is $\frac{n+1}{2}$.

2(A). Answer any two out of three. **[10]**

(1) State and prove Euler's formula for a connected planner graph.

(2)For the following graph G find maximal independent sets.



(3) If A(G) is an incidence matrix of a connected graph G with n vertices ,then prove that the rank of A(G) is (n-1).

2 (B). Answer any one out of two. **[4]**

(1) Define planar graph. Prove that complete graph with 5 vertices is non-planar.

(2) Define: Incidence Matrix. Also state any four properties (observations) of incidence matrix.

3(A). Answer any two out of three. **[10]**

(1) In usual notation prove that the bilinear transformation maps three given distinct points z_1,z_2 and z_3 onto three specified distinct points w_1,w_2 and w_3 is $\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$.

(2) Discuss the bilinear transformation W= z^2.

(3) Define: Conformal mapping. Prove that W= z^2 is conformal.

3(B). Answer any one out of two. **[4]**

(1) Prove that the transformation w = $\frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ of the z-plane into a line 4u+3=0 in w-plane.

(2) Find the mobious mapping which transforms points z = 2, i, (-2) of z-plane into w = 1, i, (-1) of w-plane.

4(A). Answer any two out of three. **[10]**

(1) State and prove Taylor's infinite series for an analytic function.

(2) State and prove Laurent's infinite series for an analytic function.

(3) In usual notation prove that cosh $(z + z^{-1}) = a_0 + \sum_{n=1}^{\infty} a_n\ (z^n + z^{-n})$ where

$a_n = \frac{1}{2\pi} \int_0^{2\pi} cosh(2cos\ \theta) cos\ n\theta\ \ d\theta.$

4(B). Answer any one out of two. **[4]**

(1) Expand $\frac{1}{z}$ in increasing power of (z-1).

(2) Expand $\frac{z}{(z-1)(z-3)}$ in Laurent's series for 0 < | z-1 | < 2.

5(A). Answer any two out of three. **[10]**

(1) State and prove Cauchy's residue theorem.

(2) Prove by using Residue Theorem $\int_0^{\infty} \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}$; a > 0.

(3) Prove by using Residue Theorem $\int_0^{\pi} \frac{d\theta}{(2+cos\theta)^2} = \frac{2\pi}{3\sqrt{3}}$.

5(B). Answer any one out of two. **[4]**

(1) Discuss the method for finding the residue of f (z) at the z_0 of the order m; m ≥ 2.

(2) Evaluate $\int_C \frac{5z-2}{z(z-1)}$ dz; C: |z| = 2.

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